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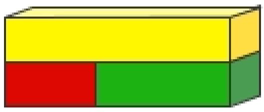
Focus on…numbers in colour

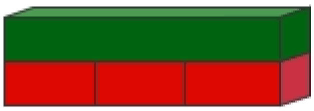
"Georges Cuisenaire showed in the early fifties that students who had been taught traditionally, and were rated 'weak', took huge strides when they shifted to using the (Cuisenaire) material. They became 'very good' at traditional arithmetic when they were allowed to manipulate the rods."
The Science of Education Part 2B: the Awareness of Mathematization by Caleb Gattegno.

Because students who cannot do arithmetic cannot function effectively in most mathematical situations that arise in life, it is worth exploring any resource that may help them. And teachers have found that Cuisenaire materials can help all learners, not only young children. They have found that when students think about and manipulate unmarked Cuisenaire rods the students use their natural thinking skills to learn to do arithmetic competently and naturally. For example, learners see in a single arrangement of rods how numbers are related at the same time by both addition and subtraction:



This arrangement reveals naturally three interchangeable ways of 'seeing' the relationship – that red + green = yellow, yellow – red = green and yellow – green = red. Students observe that no one of those three 'ways of seeing' dominates, and if you can 'see' it in one way you know that you will also be able to 'see' it in either of the other two ways.

And multiplication, division, and fraction facts, and relationships between those ideas, appear naturally to students when they think normally about an arrangement such as this:



(Three red blocks are the same length as a green block, the number of red lengths that there are in a green length is three, and a red block is one third of a green block.)

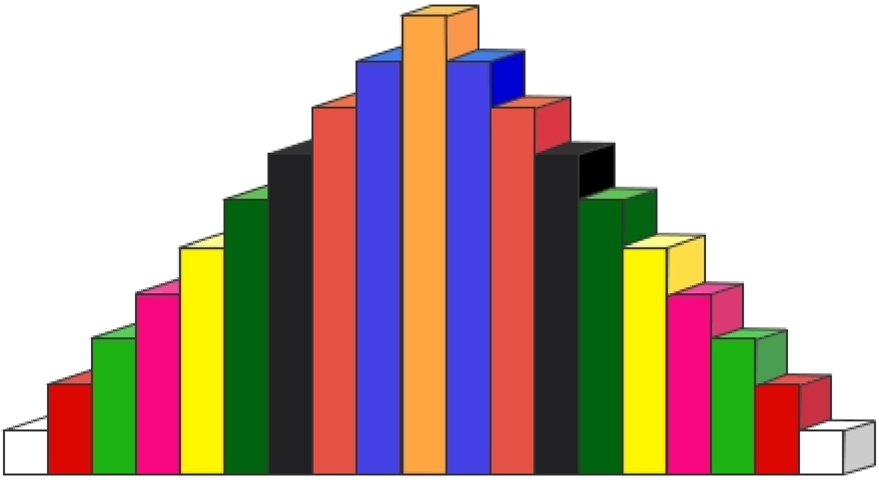
The teacher's role is to prompt students to focus on what they are naturally aware of, to help them realise how much they know naturally so that they develop confidence in their own thinking.

During 1953, Dr Caleb Gattegno saw young students in Georges Cuisenaire's classroom learning to do arithmetic competently and rapidly using only coloured rods. Many years later, Gattegno told English mathematics teachers that "Children could improve their mathematics if they worked with Cuisenaire rods instead of notation and verbiage… …there was something in the manipulation of the rods which made their mind clear about conventions and notations and other things…." (you can listen to Gattegno on the ATM website).

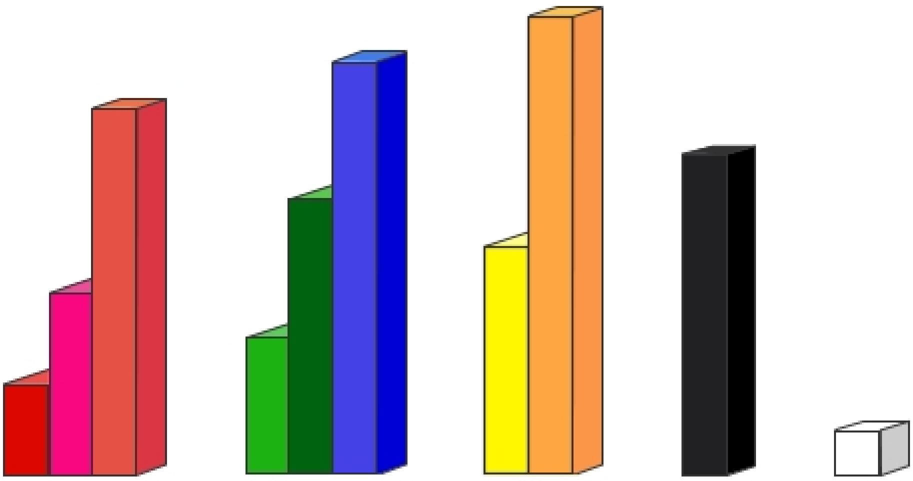
To see for yourself how Georges Cuisenaire worked with pupils using his rods (and nothing else) in an Alpine primary school, it is well worth persevering 07:18 minutes into this French film.

In Lessons With Cuisenaire Rods - Notes On The Filmstrip 'Numbers In Color', Gattegno explains that Cuisenaire rods are cuboids in white, black, red, pink, tan, light green, dark green, blue, yellow, and orange. Students should discover for themselves that only those ten colours appear.

When students make 'staircases' using all the colours they discover that the difference between consecutive rods is always the same and equal to the length of a white rod:



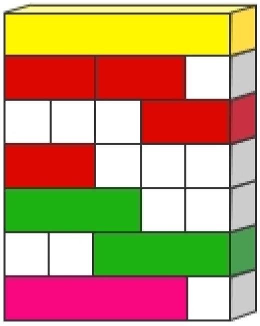
Cuisenaire deliberately chose colours for the rods such that, when students measure each rod by the white rod and then explore relationships, they can make use of, possibly unconsciously, the significance of the colour 'families' – the reds, the yellows, the blues, the black and the white:



Explorations

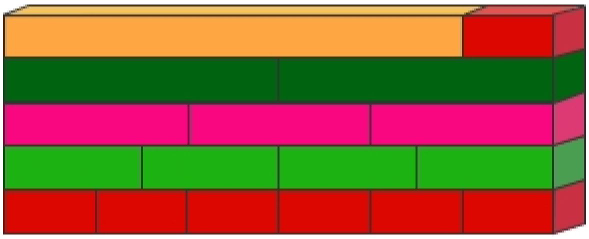
Many explorations of number facts and relationships are possible, such as the few suggested by the following arrangements of rods.

Decompositions of numbers



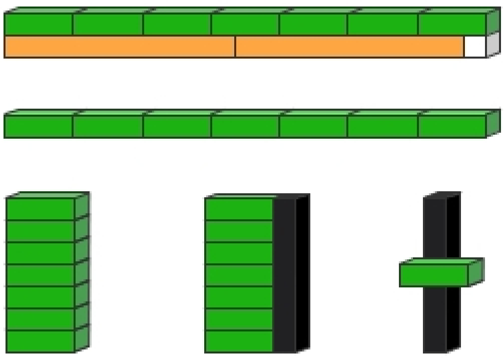
A student can read into this arrangement that 5 = 2 x 2 + 1 or 5 = 3 x 1 + 2 or 5 = 2 + 3 x 1 and so on.

Pairs of factors
What lengths can be made up of rods of one colour?



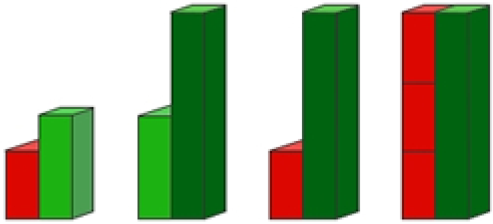
(12, made up of 10 + 2, is also made up of 2 x 6, 3 x 4, 4 x 3 and 6 x 2.)
It becomes natural to make a cross as a symbol for a composite number… the factors are the rods used in the cross.

For example, a student sees that seven light-green rods (each of which is 3 relative to the white rod) placed end to end are the same length as two orange rods (2 x 10) plus one white rod (1). When the seven light-green rods are arranged to make the other possible rectangle, a black rod (7) fits along it. Therefore it is natural to represent 21 (7 x 3) with a cross of one green and one black rod:



Prime numbers
What lengths cannot be made using rods of one colour only (excluding repetitions of the white rod) or by a chosen group of rods repeated a certain number of times?

Fractions of fractions
For example, two thirds of one half is one third:



Mental activities

You can devise mental activities to practise number relations that are of value in everyday life. For example, using a handful of rods that students are not allowed to touch:

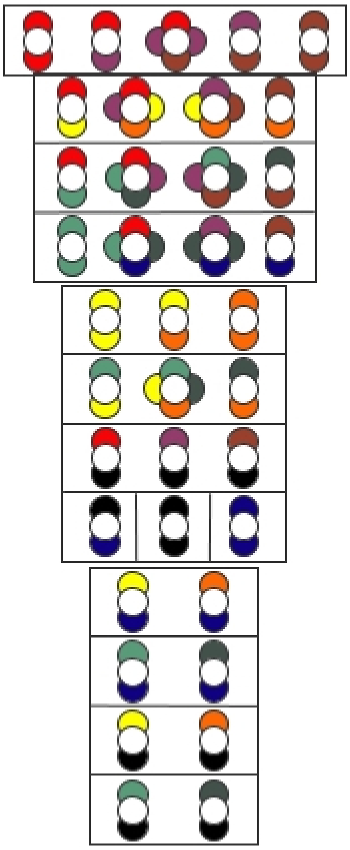
- place a handful of rods in front of students
- ask what length would be made if all the rods were placed end to end
- no-one may touch the rods; they must operate mentally.

What are students' mental methods? What do they think is the best way to proceed? Perhaps they look out for products? Or do they look for complementary numbers forming tens? Do they see other obvious combinations – such as three tan rods and one dark green rod making 30?

There will be lots of possible ways of proceeding.

The cardboard materials
To gain practice in rapid mental calculation, students can also use the 'cardboard' materials that Cuisenaire devised to complement his rods. The aim of activities involving them is for products and factors to become 'second nature' to students.

The **Product Chart** shows the 37 different products that can be made by one or two crosses of rods. But no numbers appear on the chart – the colours of the rods are used instead. A white circle that is flanked by coloured crescents represents each product. The colour of each crescent is the Cuisenaire colour of the factor it represents (so a red crescent represents the number 2). Can you work out how Cuisenaire arrived at his arrangement starting from the colour families?



The values of the products shown in the chart are:

4		**8**		**16**		**32**		**64**
	10		**20**		**40**		**80**	
	6		**12**		**24**		**48**	
	9		**18**		**36**		**72**	
		25		**50**		**100**		
		15		**30**		**60**		
		14		**28**		**56**		
		63		**49**		**81**		
			45		**90**			
			27		**54**			
			35		**70**			
			21		**42**			

The Lotto Game

Four students play the game together.

The teacher gives three 'master-cards' to the group. The 37 pictures from the Product Chart, of circles surrounded by coloured crescents, are reproduced on the master-cards, distributed randomly between the master-cards in sets of 12, 12 and 13.

A student allocates herself to each master-card, with the fourth student acting as a 'banker' who calls the numbers on counters drawn from a set of 37 counters in a bag. Each counter shows one of the 37 numbers represented on the product chart (listed above).

The banker draws the counters, one by one, from the bag, each time calling out the number.

Each player scans their master-card, and, if they spot on it the pattern showing the factors of the number called, they say 'me'. If this is correct, the student places the counter on the appropriate circle, and scores the number on the circle.

If the student whose master-card contains the factors fails to see that they have it, the banker (if she spots it herself) puts the counter on the circle upside down. If no one can find the right circle, the counter is put to one side.

The banker is responsible for checking claims, counting scores, and so on.

The cards are exchanged for each fresh game, and turns are taken at being the banker.

To encourage discussion several students could share each master-card.

In *Lessons With Cuisenaire Rods* Gattegno warns that it is unwise to call a colour by its figure name (for example, to say 'two' for red). "This creates confusion and is a waste of time."

Product Cards

The Product Cards contain the 37 'pictures' of products that are on the chart, with one product on each card. You, or your students, could devise games to play with these cards.

Gattegno describes the following game for two players in Arithmetic - A Teacher's Introduction To The Cuisenaire-Gattegno Methods Of Teaching Arithmetic:

The 37 cards are shuffled and dealt, then played alternately. Each time, the first of the two players who recognises and calls out the product, scores the value of the product. The player with the larger score wins.
Variation: each player plays their top card and the products are compared. The player with the larger product takes over the card with the smaller product (or vice versa) and scores the sum of the two products. Scores are recorded, and added up at the end.

We have introduced only a very few of the many ways in which students can learn with Cuisenaire materials. You will find more guidance about effective ways of working with the materials in The Cuisenaire Gattegno Method of Teaching Mathematics - A Course For Teachers - Volume 1 by: C. E. Chambers.

NRICH provides a Cuisenaire 'environment', but it is important to remember that Cuisenaire designed his materials for learners to handle and manipulate physically in three dimensions. You can find problems using NRICH's Cuisenaire 'environment' on the NRICH website.