Getting started with early algebra

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In these two articles, we describe how we are meeting the ambitious Key Stage 1 objectives of the new curriculum through mathematical writing. The 2013 curriculum requires learners 'to move fluently between representations of mathematical ideas.' (The National Curriculum in England, September 2013, page 3.) Children now have to study all four arithmetic operations and fractions as operators for small numbers from Year 1.

Our approach was pioneered by Dr Caleb Gattegno, founding secretary of the ATM in the 1950s. He used mathematical writing to record relationships between colour coded Cuisenaire rods. The articles can be read as an introduction to the Cuisenaire-Gattegno approach to early algebra. They form a self-contained guide, with their links, to learning and teaching.

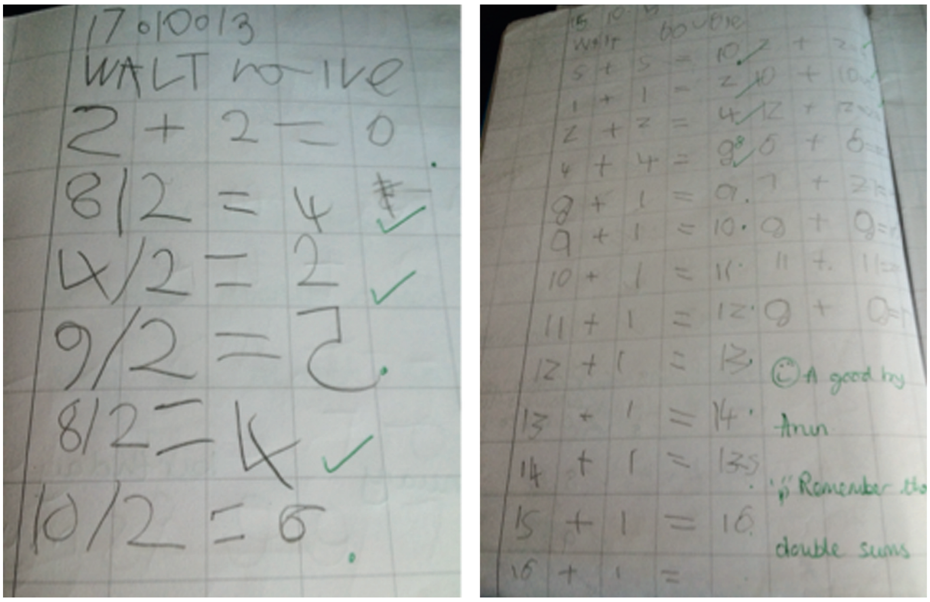
This first article describes the position at Bursted Wood, a two form entry primary school, before the new curriculum was introduced. It illustrates the approach with a discussion of the mathematical vocabulary, symbols and notation that we introduce to model addition. In the second article, 'Experiences with early algebra' we extend the discussion to the remaining operations, and fractions as operators. We give examples of the learners' work at the outset of the project and at the end of two terms.

One of the authors has been working with schools to re-introduce Gattegno's work since 2004. A report on his work has been published as a text-book (Benson, 2011) and its re-introduction to a one form entry school has been described online (Ainsworth, 2011). The other authors teach at Bursted Wood. Like many primary school teachers we were aware of the new demands that would be made of teachers and learners alike from the new curriculum: one of these being the teaching of fractions as operators in early KS1. Previously this concept was not approached until KS2, so the idea of Year 1 now needing to understand this was something that caused us concern. Teachers quickly looked to where they could find guidance in teaching this effectively and the NCETM provided this support. It was during training at an NCETM Professional Development Lead Support Programme that links were made with Ian Benson. He shared the work that he had already done and said that he was interested in developing it on a larger scale. Work started in March 2013 in breaking down the new curriculum so that it could be delivered by applying principles pioneered by Dr Gattegno.

Transition to Year 1 – supporting early mathematics

The question of how to transition children successfully from the Reception classes to Year 1 is something that teachers have always tried to plan carefully at Bursted Wood. However, striking the balance between continuous provision and formal learning was often something we felt could be improved. As a school we wanted to maintain our high expectations and support those children who were ready to make their first real steps towards formal learning. Many had been in nursery education since before their first birthday and therefore were more than ready to write, sit and listen attentively. However, at the same time we did not want to lose some of the motivation gained from practical, active learning experiences such as small world play, role play and brick play. From a mathematical point of view, we understood that many of the misconceptions coming through the school started early on and were often language based. Children simply did not have enough practical experiences through mathematics to make them confident, independent learners. Vital connections were not being made effectively.

Maths before using the Cuisenaire resources varied for each child. Although lessons were carefully planned, differentiated and often creative in nature, not all children were accessing the lessons equally. It was often true that some children, for example, could easily recite their number bonds and some times tables, but once they were taken out of context, they could not apply that knowledge. Gaps in their knowledge and understanding varied so much that it was difficult to support everyone's



Halving and Doubling with a 'counting on' strategy

needs and there appeared to already be too many strategies learnt at such a young age, but none that were efficient enough for children to make the necessary connections.

The figure shows examples of the children's work before the project started. In one exercise halving is performed by rote. In another, when asked for a table of doubles, the learner recalls the numbers where they are less than ten, and then proceeds to count. Although some children achieved correct answers, they did not use the variety of resources such as multilink cubes, straws, Numicon, counters etc well enough to be accurate all the time. The children were using far more procedures than perhaps they needed, without the conceptual understanding that we hoped they would have.

> *Children need to be both procedurally and conceptually fluent – they need to know both how and why. Children who engage in a lot of practice without understanding what they are doing often forget, or remember incorrectly, those procedures. Further, there is growing evidence that once students have memorised and practised procedures without understanding, they have difficulty learning later to bring meaning to their work*
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> (Stigler, Hiebert, 1999).

Fluency is a key word in the 2014 National Curriculum and finding a way of making our students fluent in maths was something we wanted to achieve as a school without making the mathematical diet too rigid and formal.

Russell (2000) suggests that fluency consists of three elements:

Efficiency this implies that children do not get bogged down in too many steps or lose track of the logic of the strategy. An efficient strategy is one that the student can carry out easily, keeping track of sub-problems and making use of intermediate results to solve the problem.

Accuracy depends on several aspects of the problem-solving process, among them careful recording, knowledge of number facts and other important number relationships, and double-checking results.

Flexibility requires the knowledge of more than one approach to solving a particular kind of problem, such as two-digit multiplication. Students need to be flexible in order to choose an appropriate strategy for the numbers involved, and also be able to use one method to solve a problem and another method to check the results.

The children, at the outset of the project, did not have this sense of fluency and more importantly they did not have a grasp of the language needed to make mathematical connections. From a teacher's point of view, it was hoped that the Cuisenaire rods would start each child off equally with the same resource, where everyone could access the same lesson and provide tasks that could promote a positive maths culture within the classroom.

Introducing the Cuisenaire-Gattegno approach

Gattegno believed that 'rather than teach mathematics we should strive to make people into mathematicians.'

(Gattegno 1974, p 82). Mathematicians use algebra to represent and reason about relationships between mathematical objects and actions. Gattegno showed that infants can master algebraic expressions using all four operations and fractions as operators. He did this by developing a colour coding system for Cuisenaire rods and using these expressions and equations to name patterns made with the rods.

Gattegno argued that the traditional approach to arithmetic, based on counting forwards and backwards, left too many children with an inadequate understanding. In his approach Cuisenaire rods are used to model arithmetic with integers (whole numbers) and rational (mixed) numbers. Number names for the rods are not introduced until their relationship to one another has been fully explored symbolically.

In the Cuisenaire-Gattegno approach learners speak about and write mathematics as a language. They recognise mathematical activity as the unfolding of concepts that they approach from four distinct perspectives. These are *Actions* (using sets of fingers or coloured rods), *Behaviour* (using imagery generated by these activities), *Speaking* (using language to describe these images) and *Writing* (using symbols and notation).

At the beginning of the 2013 Autumn term, Cuisenaire rods and Gattegno's teachings were taught alongside a more traditional primary maths format in Year 1. Rod lessons generally took place twice a week, outside the standard hour long daily maths lessons. We followed Gattegno's first

textbook (Gattegno, 1963) strictly from the very beginning and initially we did not mix the National Curriculum objectives with the outcomes in the books. We were essentially using the rods to backup the 'normal' maths lessons.

To start with, the children encountered the rods through 'Free Play'. In the early exercises of free play with the rods, learners observe important characteristics of the set of rods which we express as propositions, that is

- rods of the same length are the same colour (and vice versa)
- rods of different colours are different lengths (and vice versa)
- the length of every rod can be made by a train of white rods.

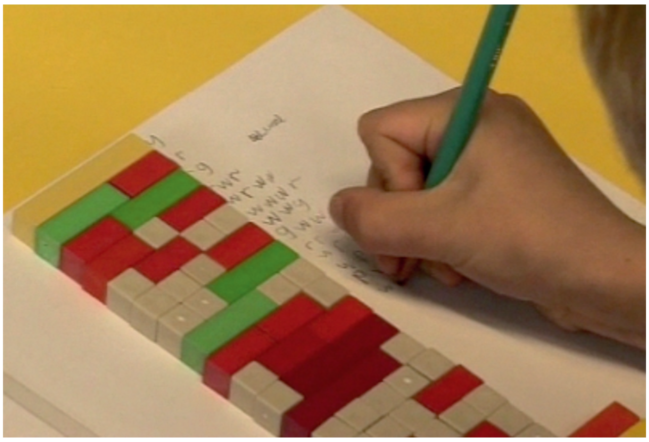
Gattegno insists on using appropriate mathematical language from the very beginning. At first some of the language felt alien to a young Year 1 classroom, especially from the point of view of a teacher. It would have been tempting to make some of the terminology child friendly, but with academic support from Dr Benson and his experience with Gattegno's method, we remained faithful to the vocabulary. Very soon terms such as 'equation' and 'equivalent' became a natural part of the Year 1 vocabulary. The children were certainly never uncomfortable using such terms. They were happy talking through what they could see and make and it became a common language which everyone was accessing. It progressed quickly into a formal written language too, which many were happy to express in 'free writing' sessions. It was the beginning of the realisation that the children were not afraid of the formal.

One of the first mathematical terms that the children encounter is 'equivalent'. They use the rods to form 'trains' (a term invented by Gattegno) by putting them end by end. Underneath each train the children make trains of equivalent length and begin to form 'patterns.' We took the rods in turn and studied them for patterns of equivalence before moving on to write equations that named the pattern.

The children quickly grasped the letter codes for the coloured rods (orange, 'o' is longest and white, 'w' is smallest) and became confident at expressing what they could see. As a class, we came up with a saying:

'say it, make it, solve it'

This in turn helped the children to read through what they could see in the equation, progressing to making



what they had read before using that information to solve a problem or calculation. Working through this process helped their vocabulary because we were focusing on the equation and what it was asking us to do in terms of the rods we needed to select rather than just an answer. Children's understanding of instructions and positional language also improved and lessons were engaging and fast paced.

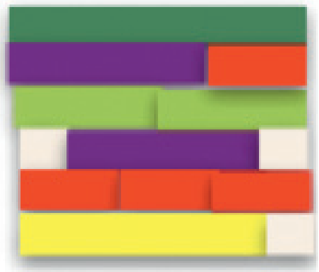
Introducing signs and symbols

Gattegno argues that children have no difficulty disambiguating names. That is, they can use the same name to refer to many things, and they know that someone can have many names. For example, they understand that several children can have the name 'George,' and 'Uncle Harold' is also 'Dad's brother.' The teacher can build on these awarenesses to achieve an understanding of the mathematical terms for actions such as plus or minus.

Infant teachers themselves know best how to demonstrate a word like 'plus' in concrete terms, other than with the rods. We have derived our approach from one suggested by Madelaine Goutard and Australian educationists (Goutard 1964, NSW 1976). The Australians first dramatise situations implied by concise statements such as:

- a boy with shoes *plus* socks
- a girl with hat *plus* raincoat
- inside this backpack is an apple *plus* a sandwich

Then, after a series of such dramatisations in successive non-Cuisenaire lessons, learners carry out the plus operation in the construction of a pattern of a rod and a train of equivalent length. If it is for the dark green (code 'd'), the they would put down a purple and then accompany the action with the words 'purple plus red.' Similarly for other trains in the pattern in the illustration.



A pattern for the dark green rod

After a few similar lessons for consolidation, the real meaning of the operation of addition can be seen and understood. That is, we choose one end or the other to add a new car to a train and regard the collection of train cars as one whole.

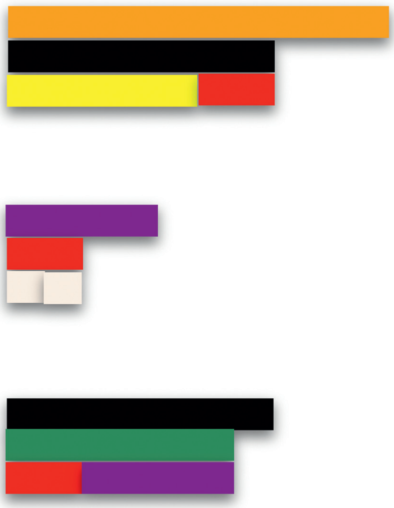
This idea of seeing groups and patterns helped the children move away from the inaccurate counting that had previously hindered their successes. If a child had been presented with an addition sum, they would have used bricks, fingers or counters to make each individual number and then count the total. Often when counting the total, they had forgotten the original set of numbers they were using and would end up very muddled and without a strategy to recover. Instead, the Cuisenaire rods allowed the children to see the whole groups they are bringing together and the new whole group it makes.

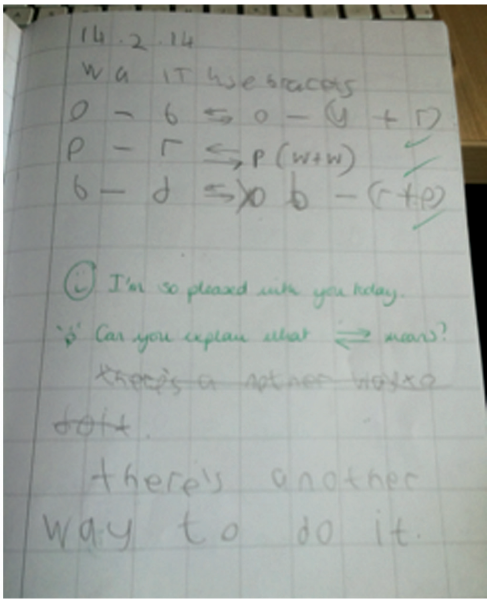
The same could be said for using subtraction. Instead of 'taking away', we used the rods as Gattegno suggests to find differences. The children could confidently compare two rods against each other and describe the comparisons using $<$ and $>$. Once the children had mastered that particular vocabulary, they easily moved on to filling the difference with the missing rod.

An interesting change happened in the class with the = sign. Previously it simply meant 'answer' to the children. Often we just felt that the children knew they had put something at the end of their equation, but were not reading what the equation was asking them to do. By using the word equivalent as the meaning of the = sign, children began to understand the relationship between the letters in an equation. The children were also happy to work with other signs, such as 'harpoons' to express what they could see happening to a written expression when brackets were added or removed.

Conclusion

We have set out an approach to integrating Cuisenaire-Gattegno mathematics with the Programmes of Study for Year 1 of the 2013 curriculum. We found that rod lessons were much





Studying brackets: introducing the 'harpoons' sign for re-writing an expression

more investigational, conversational and often child led compared to other maths lessons. The low threshold, high ceiling nature of the resource allowed for quite sophisticated learning to take place, which had some of the greatest impact on the middle to less able learners.

Using the vocabulary of equivalence, plus and difference helped to develop other mathematical strands such as measures. Children could use their understanding of the terms to carry out investigations independently and accurately record what they were seeing using notation similar to their writing with the rods.

In the next article we discuss how to introduce the meaning of 'equivalence' and 'minus,' the operation of subtraction, and reading and writing expressions for multiplication and fractions as operators. We give examples of the learners' work after two term's experience.

Readers interested in learning more will find links to virtual Cuisenaire rods and Gattegno's textbook for children, 'Mathematics with Numbers in Colour' at **sociality.net/imagery.**

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