On the awareness of patterns of Cuisenaire rods[1]—*Ian Benson*

This note discusses four key patterns identified by Gattegno in the initial chapters of his 55-concept curriculum (the Cui curriculum) for elementary school mathematics (Gattegno, 1974, p.124). It links to two activities to study patterns of Cuisenaire rods that can be carried out online using the *Virtual Rod Resources*.

Cui poses students a series of questions concerning configurations of colour-coded rods, so that students become aware that they can move freely between the following (op. cit., p.40):

1. imagining an activity of putting rods end to end (to make a train) or side by side (e.g., a staircase)
2. recognising that there are many ways to choose a pair of colours to combine
3. simultaneously perceiving a train and its component rods, and
4. understanding that the written sum is both inherent in the train and distinct from it.

Here are two activities – try them yourself and monitor how your own awareness moves between the four descriptions above.

ACTIVITY 1 START

Ask learners to work with one rod (of any length). Beneath it, ask them to make the same length using other rods. Tell the learners to distinguish between permutations of different colours, but not between permutations of the same colour.

How many different ways can you make the same length?

How could you be organised?

What are the names of the patterns that you make if you use the colour codes (*w*,*r*,*g*,*p*,*y*,*d*,*k*,*n*,*e*,*o*) and paper train names (*wrg* etc) for the written sum?

ACTIVITY 1 END

ACTIVITY 2 START

This is a Key Stage 3 activity. It can be found at *http://nrich.maths.org/4338*

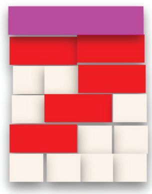

Figure 17: Equivalent trains of rods

Ask learners to work with one rod of each colour and make trains equivalent to it in length using white and red rods only.

The figure shows the five different ways to make the length of the pink rod. Tell the learners to count white, white, red and white, red, white as different, even though they both use two white rods and one red rod.

Ask the learners, "using your rods, can you work out how many different ways there are, using only the red and white rods, to make up:

- The white rod?
- The red rod?
- The light green rod?
- The yellow rod?
- The dark green rod?"

In each case, ask them to make a list of all the different ways using the letter codes and paper train notation. That is, the figure is encoded [*p*,*rr*,*wwr*,*rww*,*wwww*]

Ask if they can spot a pattern that would help them to predict how many different ways there are to make up the black rod using only red and white rods? Ask them to check their answer for the black rod.

Without using their rods, ask the learners how many different ways are there to make up the orange rod, and ask them to explain the pattern.

ACTIVITY 2 END

These exercises encourage systematic reasoning about the relationships between the whole numbers. Originally proposed by Gattegno and developed by NRICH, they demonstrate a power to engage learners at any age, and at every level of proficiency. The code names for the rods and trains help learners to record and enumerate the set of possible permutations. This is needed when their box of rods is exhausted, or the diagram becomes too large for the canvas. The symbolic representation can be manipulated using a modern functional programming language. This suggests further exercises to write algorithms to generate the sets of train names, as envisaged in the statutory programmes of study for computer science.

In 1974, in 'Common Sense of Teaching Mathematics', Gattegno includes as an illustration a diagram of a set of complete patterns that he uses to define what he means by an integer. He writes, "the set of all ways of making a particular length with rods produces an equivalence class: all trains in the set are equivalent since they have the same length" (Gattegno, 1974, p.51). *Figure 18* reproduces his drawing, with annotations to show the correspondences they suggest.

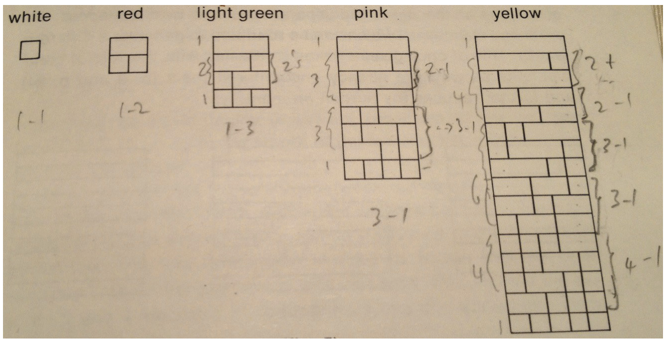

Figure 18. A set of complete patterns of rods from white to yellow

The diagram arranges the trains into product groups, that is, groups of trains with the same number of rods. Looking at the number of trains in each group we see a pattern emerge corresponding to the coefficients in Newton's Binomial expansion or the cells in Pascal's Triangle: 1, 1-1, 1-2-1, 1-3-3-1 etc.

In developing his thinking about how to introduce early algebra and number systems Gattegno proposes the study of four patterns that draw on some, but not all, of the trains in an integer. A pattern of rods, is a set of trains set side by side of equivalent length (op. cit., p.124).

I have labelled one of Gattegno's patterns a 'Ferrer' (after Ferrer's diagram which is a way of visualising the partitions of the integers). In this pattern one train is chosen to represent all the permutations for a combination of cars. The rods in my chosen train do not get any bigger as you move from left to right. I have found that learners spontaneously develop a method like this to represent a set of permutations for a combination both as a train (if they have the rods), and in writing if they run out of rods. Using this encoding of the complete pattern learners have been able to enumerate all 128 trains in the complete pattern for the brown rod (equivalent to 8 whites).

Gattegno identifies three other key patterns:

1. Measure – a pattern showing that a length of a train L is equivalent to nw white rods ($L \sim nw$, where n is an integer multiplier – a numeral used as an adjective, w is the code for a white)
2. Complements – a pattern consisting of just those two rod trains of equivalent length to a rod, and
3. Iterates – a pattern arranged as in the Ferrer case, but with either all the cars the same colour or all but the last being the same colour.

The first activity has been given to learners in Years 1-7. Working in pairs, learners as young as 6 have been able to create complete patterns for dark green (32 trains). One learner in Year 7 enumerated the pattern for brown (128 trains). Teachers found a high correlation between performance at these tasks and the ability to master the four operations and fractions as operators for small numbers. (Benson, 2011, pp.41,70) This is now an explicit aim of the 2014 mathematics curriculum for Year 1.

The second activity was set as an exercise to 20 Year 7 students. They had been exposed to ten hours of lessons, which introduced the rods, Gattegno's early algebra exercises (Gattegno, 1970, p.17) and list processing concepts. While no learners were able to solve the problem, all attempted the early questions correctly. Most looked for a pattern suggested by the sequence of questions, rather than the sequence of rods. List processing notation enabled the learners to explain their work and highlighted those who were able to work systematically. It was a useful formative assessment activity.