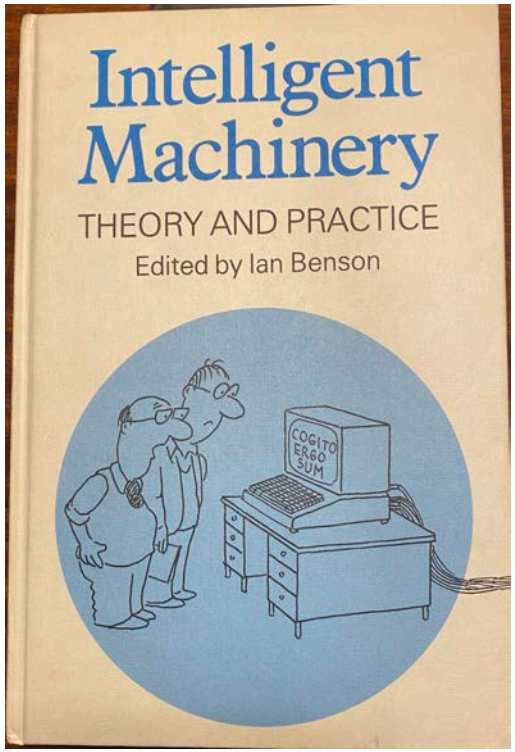
An AI Future?

"We must ensure that young people are equipped to shape an increasingly AI-powered world. They need to be able to navigate misinformation and other challenges, and they also need to be able to take the opportunities that will be available to those who can become the most skilful shapers and operators of AI.

"This requires a strong focus on maths, but also the development of sophisticated analytical skills, and higher order domain-specific problem-solving ability, rooted in secure knowledge." (Francis Curriculum and Assessment Review, Interim Report)



bit.ly/IntelligentMachinery

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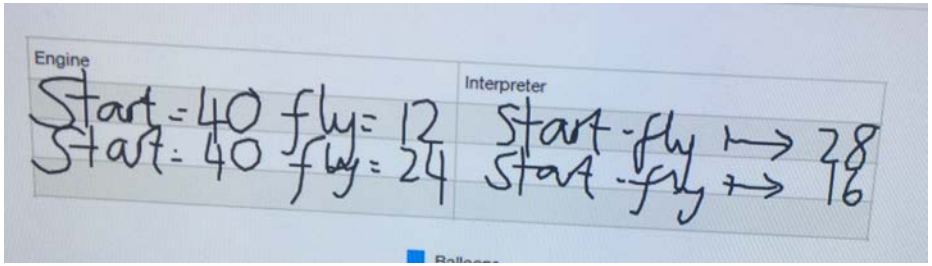


1.1 A GAI framework for mathematics assessment

"It is proposed that the critical feature for promoting algebraic thinking is not the tasks given to learners,

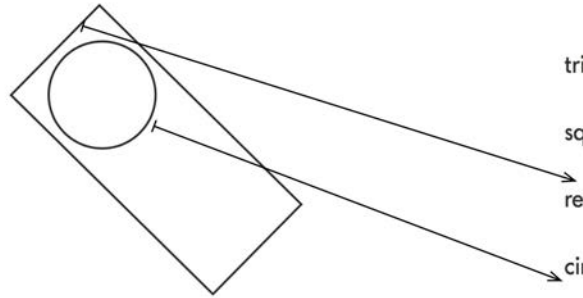
- but rather the opportunities noticed by teachers for calling upon learners' powers to express and manipulate generalities, and
- that this is enriched when teachers engage in similar tasks at their own level, so as to sensitize themselves to pedagogic opportunities when working with learners" (Mason, 2018).

John Mason (2018) How Early Is Too Early for Thinking Algebraically? https://bit.ly/Mason2018

1.2 A modern foundation for mathematics

Ordered set → Unordered set

7



Tick **two**.

- triangle ☐
- square ☐
- rectangle ☐
- circle ☐
- hexagon ☐

Tick the names of the **two** shapes in this picture.

- The concept of a function is central to this symbiosis of mathematics and informatics (or computing).
- A function is a map from a set called a domain to a set called a co-domain.
  - This is illustrated here using a curved tail arrow at the level of the sets (sometimes called an external diagram) and w
  - ith the two barred arrows at the level of individual elements of the sets (the internal diagram).

1.3 A GAI model for assessment: noticing

- Statistical prediction is an innate mental power.
- We need it to learn to speak and to interact with a world of animate objects.
- In this video a seven month old child catches a ball. What do you see?

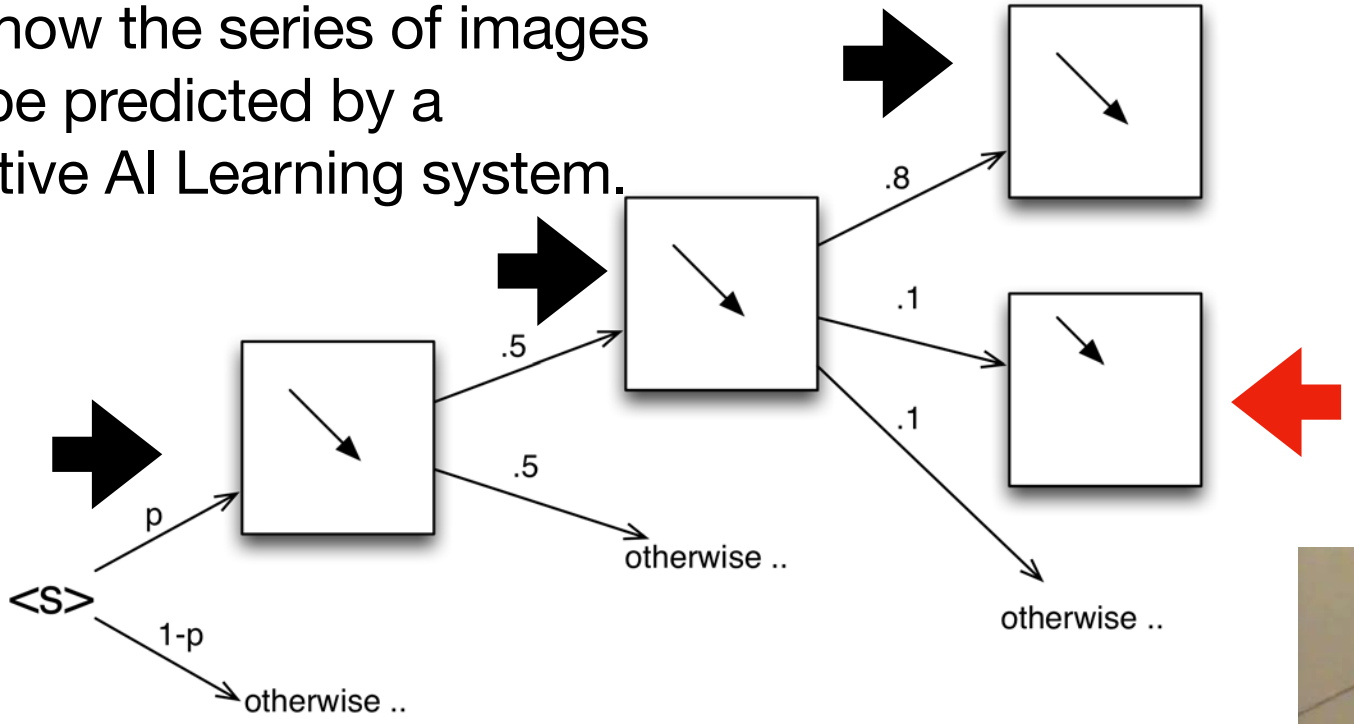
Claus von Hofsten (2007), Motor Development from an action perspective. Encyclopedia of Infant and Early Childhood Development, ed M.M.Haith and J.B. Benson

1.3 A GAI model for assessment: noticing

- Watch the end of clip again in slow motion.
- The first two transits of the ball ended in front of the baby, but the third one stopped short.
- The baby however reaches out and grasps a virtual ball at the place that it would have reached.

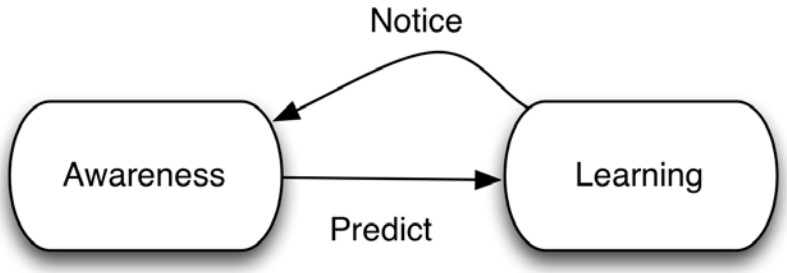
1.3 A GAI model for assessment: prediction

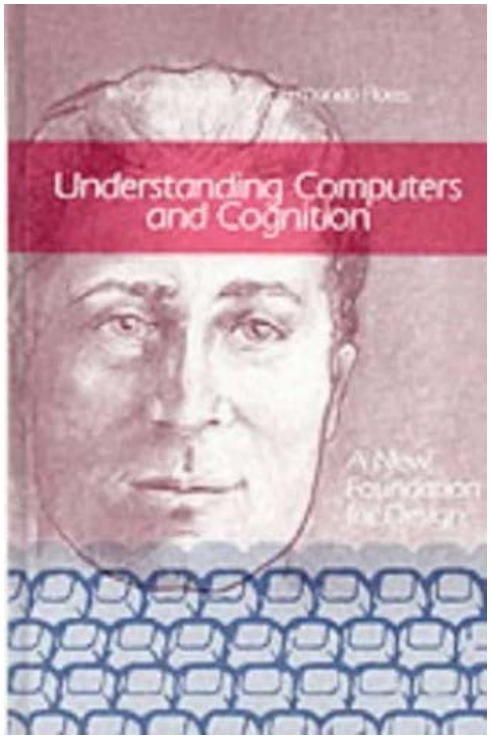
- This weighted decision tree shows how the series of images would be predicted by a Generative AI Learning system.



1.3 Breakdowns prefigure awareness

- Although it is not guaranteed learning often takes place when we notice that our predictions fail.
- Winograd and Flores proposed that this insight is used as a new foundation for the design of software systems





Winograd and Flores (1986), Understanding Computers and Cognition

2 Gattegno Mathematics (GM)

"What is it, then, that will allow us to teach mathematics to anyone with a functioning mind and an inclination to learn? Simply, finding a way to make the learner aware of the powers of his mind–the powers he uses every day, those which allowed him to learn his native language and to use imagery and symbolism. This means that the job of teaching is one of bringing about self-awareness in learners through whatever means are available in the environment: words, actions, perceptions of transformations, one's fingers, one's language, one's memory, one's games, one's symbolisms, one's inner and outer wealth of perceived relationships, and so on" (Gattegno 1974)

Caleb Gattegno (1974), The common sense of teaching mathematics.

2 Gattegno Mathematics (GM)

“There is nothing so strongly motivating as realizing you can do something that is valued and valuable. The exercise of your own powers, independently, is a major source of pleasure for human beings, whereas dependency on others breeds discontent. Learners can become frustrated when the teacher or text usurps their role by doing things for them: roles they are on the edge of assuming or actions they are already able to do for themselves. All too often learners decide that their powers are not wanted in the mathematics classroom, and so they stop using them even where there is an opportunity. It is highly de-motivating and disempowering to find that your own powers are not called upon, not encouraged, not used; it is a source of pleasure and empowerment to find that you can use your own powers to make sense of phenomena, situations and ideas. Furthermore, the more you are called upon to use your powers, the more developed and sophisticated they are likely to become; the less they are called upon, the more likely they are to atrophy, or at least to be parked at the classroom door.” (Mason 2008)

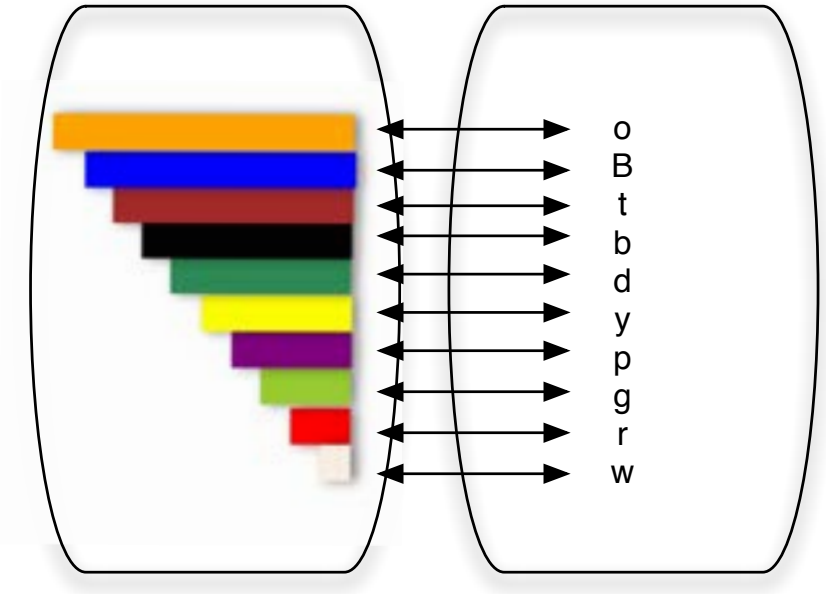
John Mason (2008), From Concept Images to Pedagogic Structure for a mathematical topic." Making the Connection: Research and Teaching in Undergraduate Mathematics Education Carlson and Rasmussen (eds)

2 GM augmented with Haskell

- The second case study is a learning episode with a student in the summer of Year 2.
- The learner had covered GM books 1 and 2, and 10 units of Haskell programming

Done New File.hs:5

```
5 data Colour = NaC | White | Red |
  Green |  Purple | Yellow | DarkGreen
  | Black | Brown | Blue | Orange
  deriving (Show,  Eq, Ord, Enum)
6
```

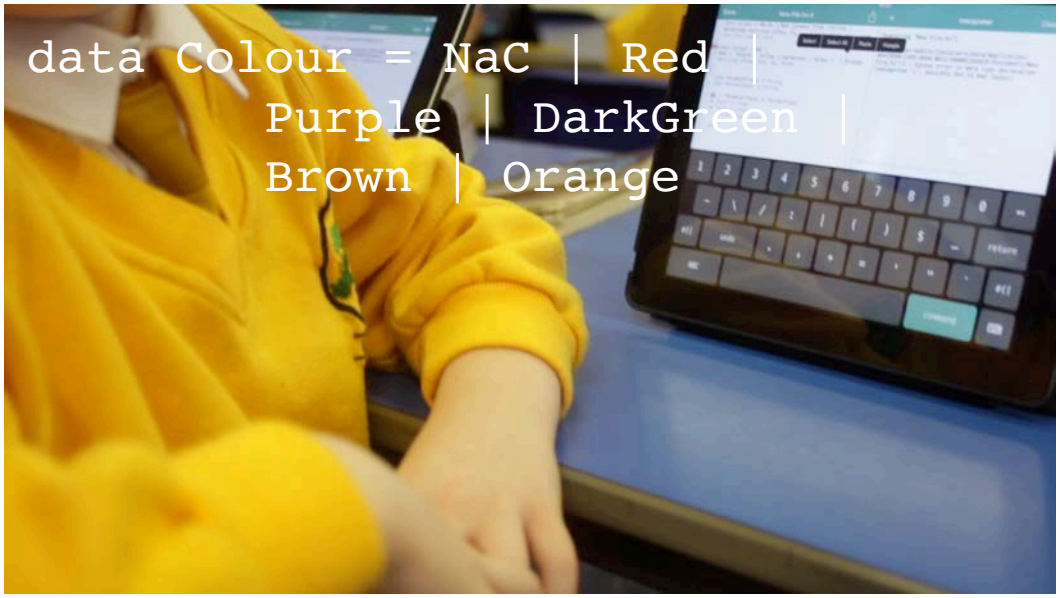


Staircase Letter Codes (GM)

Benson and Cane, Using Haskell with 5-7 year olds, Hello World, https://stanford.io/3IPGCSa

sociality.com

2.1 A GAI framework for assessment: noticing



https://youtu.be/IWEjxsDCJPg?si=5wysN5Mtfr1_3AhR&t=1532

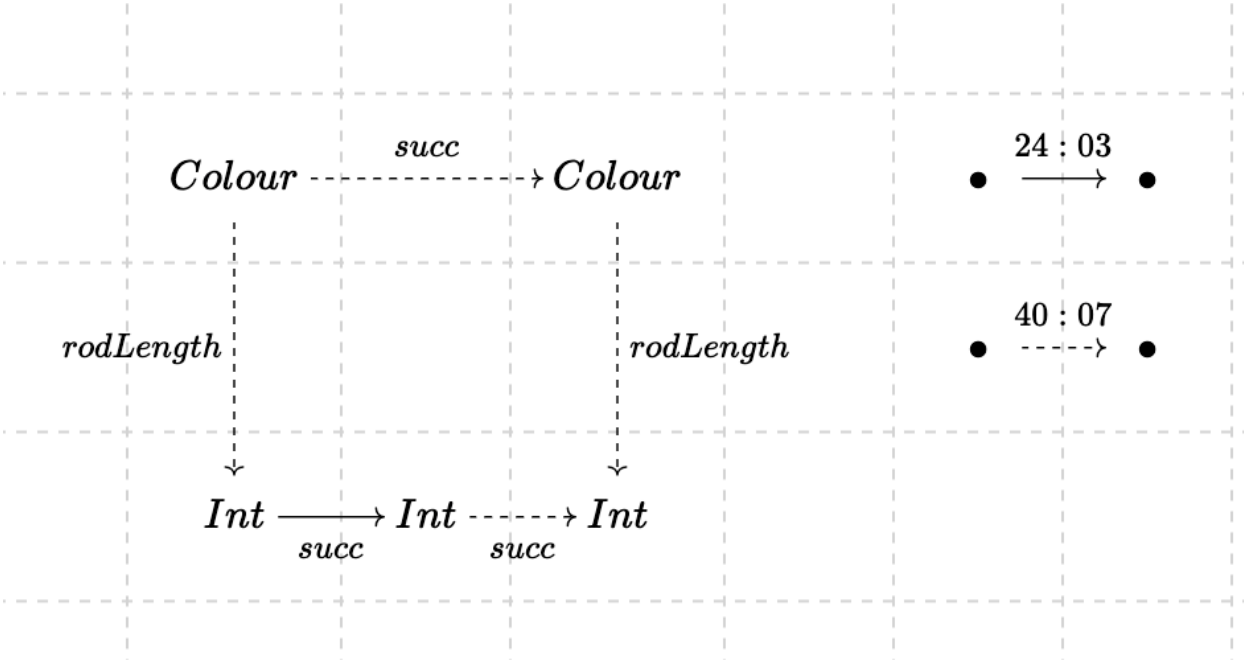
- In this second example we will study a conversation with a Year 2 student in the summer term, after two years of following the Cuisenaire-Gattegno mathematics curriculum, followed by 10 lessons programming with Haskell.
- We will see how a subtle change in the situation with a named function whose “polymorphic” behaviour is context dependent leads to self-correction by the student.

2.1 A GAI framework for assessment: abstraction

```
13:08 Q: So what are you doing? Are you doing |succ| or the other one?
19:08 A: I am doing |succ|
21:11 Q: What does that mean?
24:03 A: It means that if I do |succ|, you add one number on ... If I had six
and I did |succ| it equals (pause for reflection) 7
37:00 Q: You're right. So you do it ....
40:07 A: (speaks over Q) Actually 8 because there are no odd.
46:03 Q: Ah. You are just doing the evens.
48:06 Q: Right, Brilliant
```

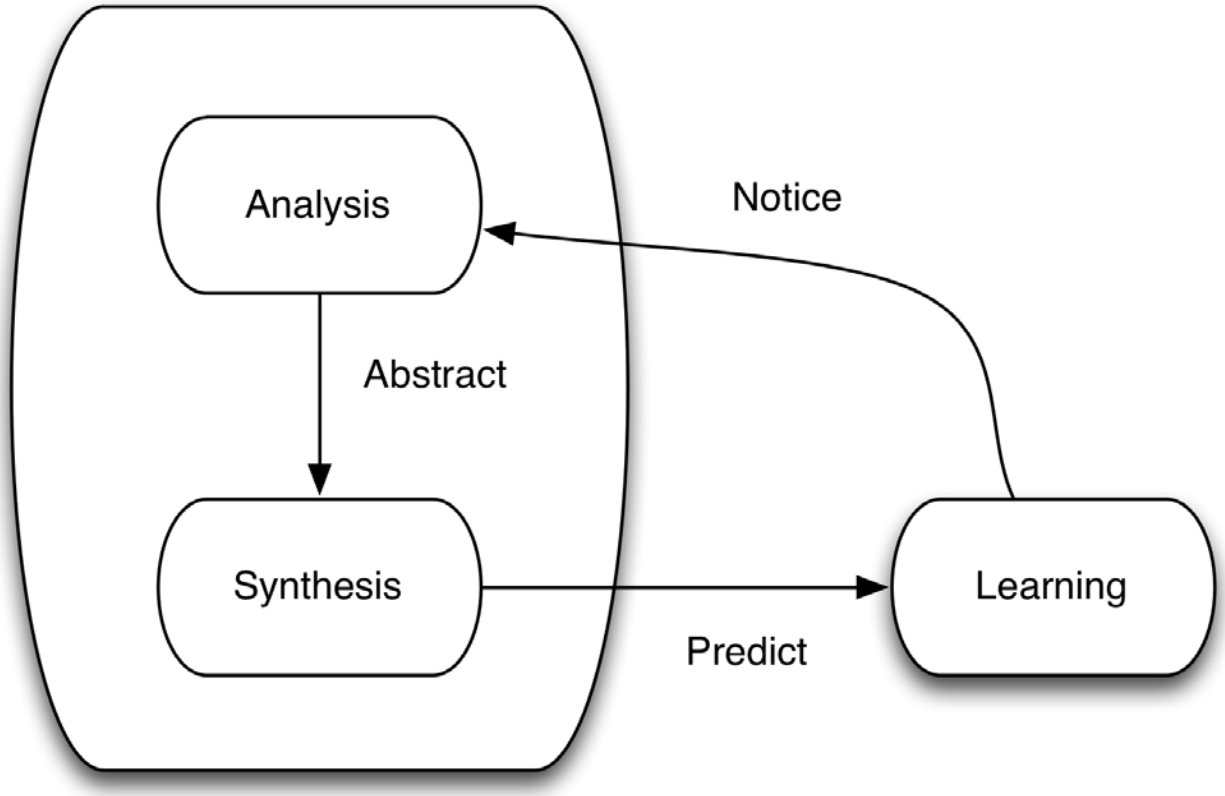
Benson and Cane (2017) , Using Haskell with 5- to 7- year olds, Hello World https://stanford.io/3IPGCSa

2.1 Noticing prefigures awareness

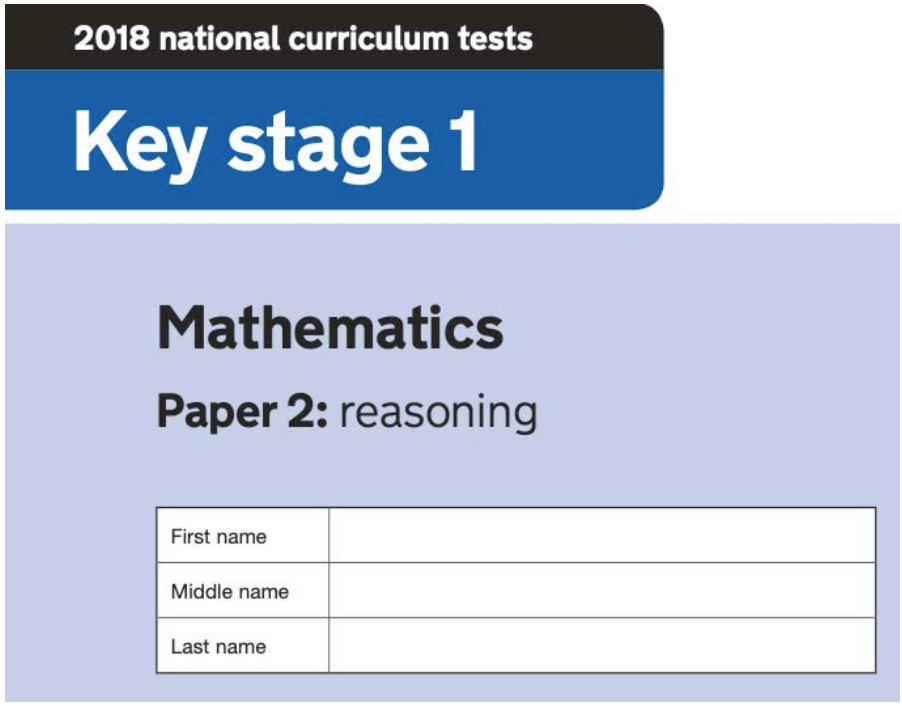


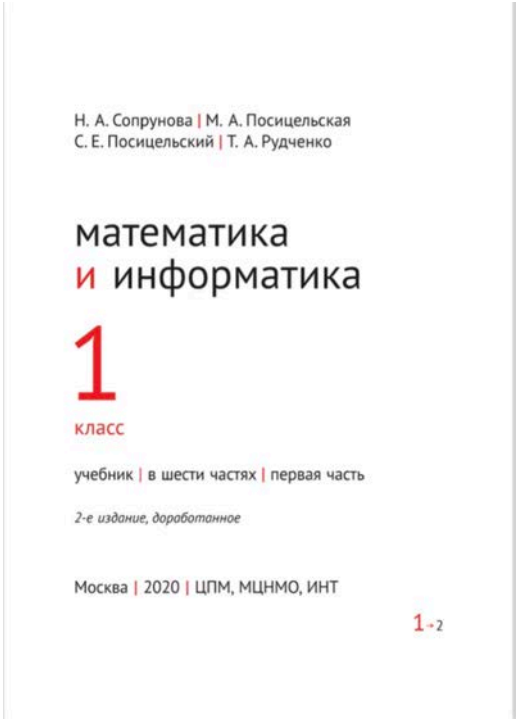
The solid arrow is his initial thought (at 24:03)
The dotted arrows represents his corrected thoughts (16 seconds later).

2.2 A GAI framework for assessment



3.1 Augmenting National Assessments





sociality.com

3.2 Augmenting National Assessments

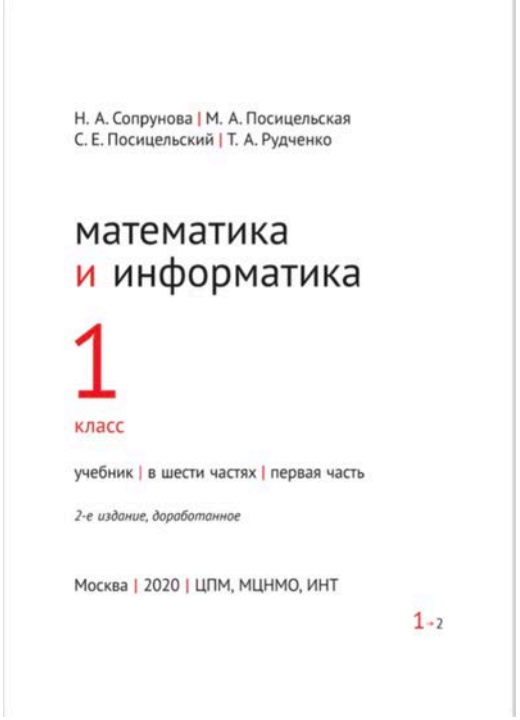
Not just numbers

In the textbook we study properties of visual objects:
strings, bags, tables, cycles, trees, etc.

Working with these objects students develop their ability:

to reason mathematically and

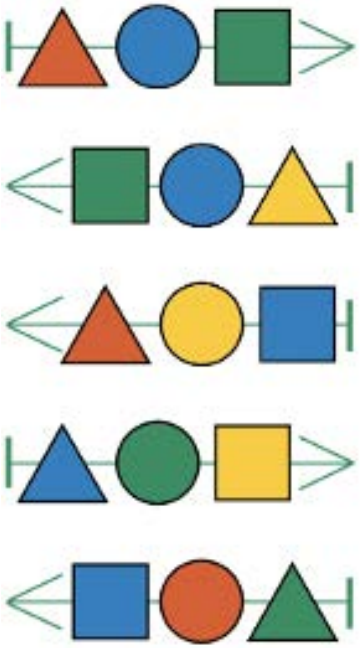
to solve problems that often do not look familiar.

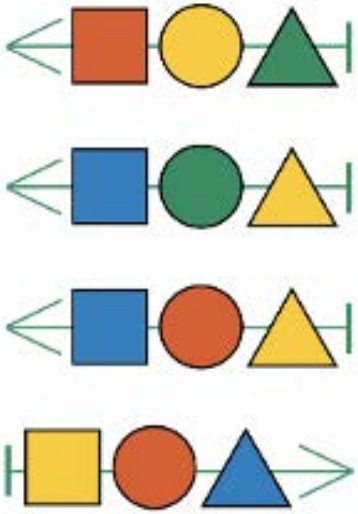


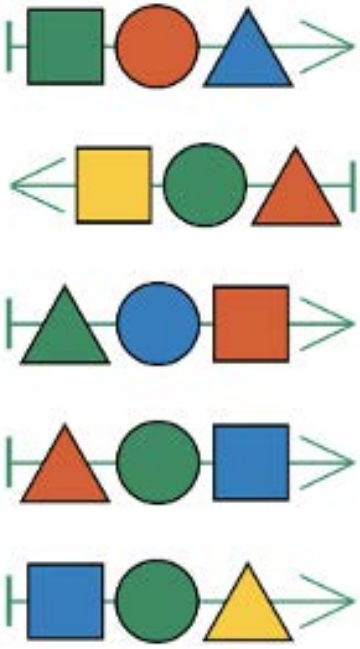
3.2 Strings (identity)

Here is a bead chain: 

Circle an identical chain in blue.

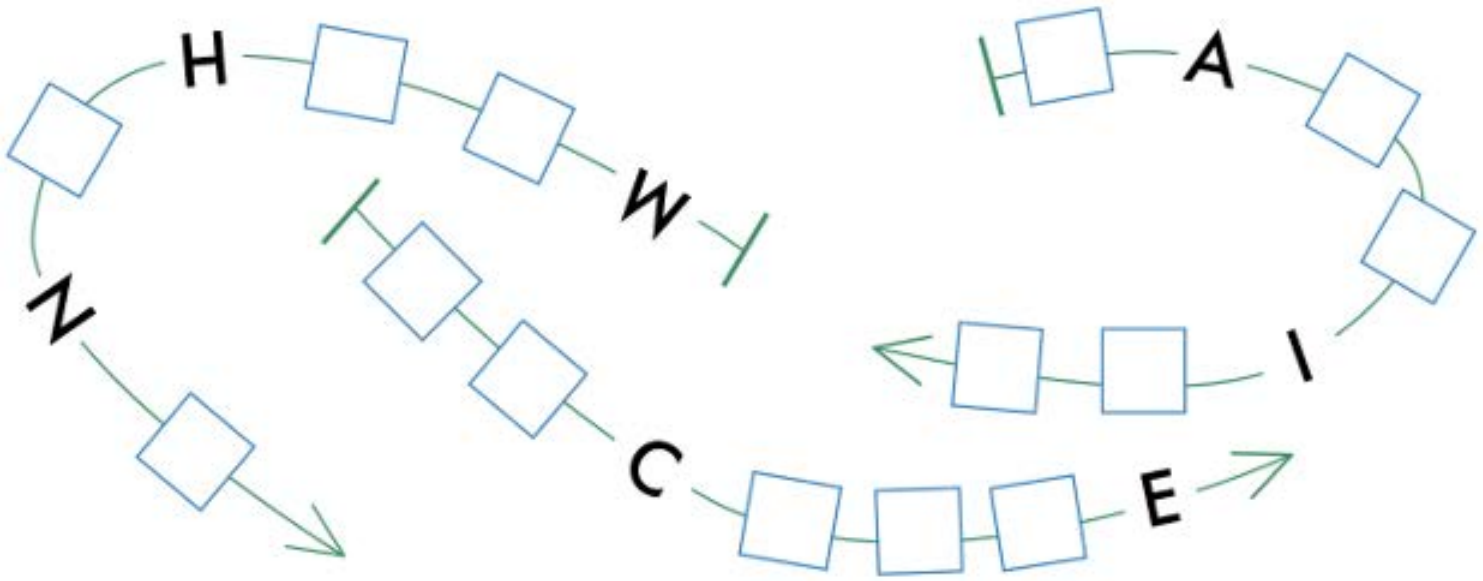






3.2 Strings (superposition)

Write down letters in the windows in such a way that these three chains are identical.



3.2 Vectors and classification

$$\sqrt{\sum_{i=1}^{8}(x_i - y_i)^2}$$

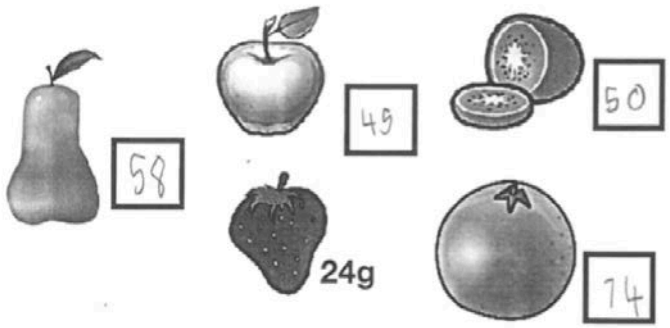
208 Fill two tables for the bag with mice.

Mice in blue skirts	
Mice in yellow skirts	
Mice in red skirts	
Mice in green skirts	

Mice in blue T-shirts	
Mice in yellow T-shirts	
Mice in red T-shirts	
Mice in green T-shirts	

- The picture shows a bag of mice wearing coloured skirts and coloured Tshirts.
  - We can classify the bag by sorting by skirt colour and by t-shirt colour.
- Each property is represented by a table (vector) with a row for each colour.
- The bag is represented by a point in 8 dimensional vector space.
- The similarity between two bags **x** and **y** can be quantified as the euclidean distance between their two points.

3.3 Making an object of a chain of reasoning



Fruits	Total weight
Strawberry & Apple	69g
Strawberry & Pear	82g
Strawberry & orange	98g
Strawberry & Kiwi	74g

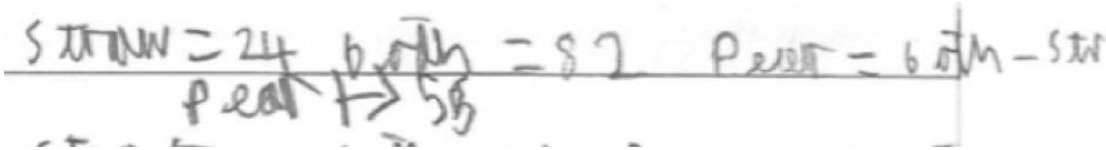
Strawberry & Apple

straw=24 both=69 apple=both-straw

Strawberry & Pear

Strawberry & orange

Strawberry & Kiwi



- Finally we have found that chains of reasoning, once abstracted in Haskell, can be generalised.
- A semantic tableaux makes an object of a chain of reasoning.
  - The preconditions shown above the line are the contents of the iHaskell cell with the data analysis.
  - The prompt shown with a barred arrow gives the result of the calculation.

https://youtu.be/IWEjxsDCJPg?si=TaXg0MXxP-ixeeRT&t=1448

4 Next Steps

Horizon mathematics/informatics - what's the same? what's different?
report https://bit.ly/CTM24Full
video https://youtu.be/C40f_0bfd4w

Gattegno (1974), Common Sense of Teaching Mathematics

Winograd and Flores (1986), Understanding Computers and Cognition

Benson (2011), The Primary Mathematics: Lessons from the Gattegno School

Young and Messum (2012), How we learn and how we should be taught. An introduction to the work of Caleb Gattegno

Cheng, How to Bake Pi (2016). Joy of Abstraction (2022)

Benson and Singer (2024), Future Learn Self-study course, Haskell Road to Mathematics, University of Glasgow/Sociality

University of Glasgow

https://www.chu.cam.ac.uk/news-and-events/people/educational-futures-outreach-activities-at-churchill/

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